



A Semi-Differentiated Model for the Potential-Sweep Voltammograms of Electrochemical Deposition Reactions

ECS 243rd Meeting - Molten Salts (High Temperature) Deposition and Extraction of Metals

June 1st, 2023

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Brigham Young University

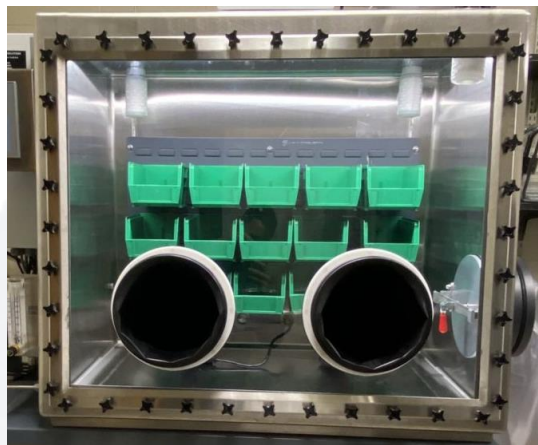
Acknowledgements



The PyRO Lab at BYU (pyro.byu.edu)



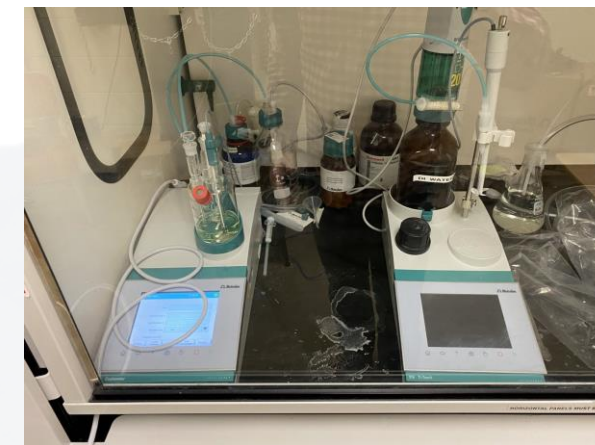
2 triple workstation inert atmosphere gloveboxes



Negative pressure workstation for transuranic compounds



Vacuum drying ovens in and out of glovebox (up to 300 °C)



Autotitrator and KF titrator for oxide and moisture analysis



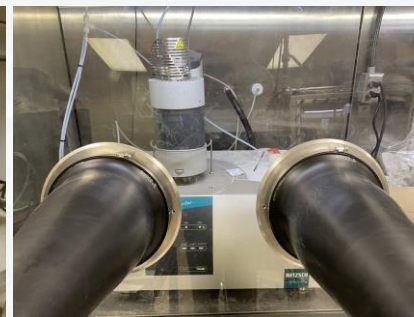
Furnaces & induction heater
max. temp. 1000-2000 °C



Potentiostats & power
supplies (≤ 24 A)



Gas analyzer (≥ 100 ppb)
plumbed to glovebox



Simultaneous thermal
analyzer in glovebox

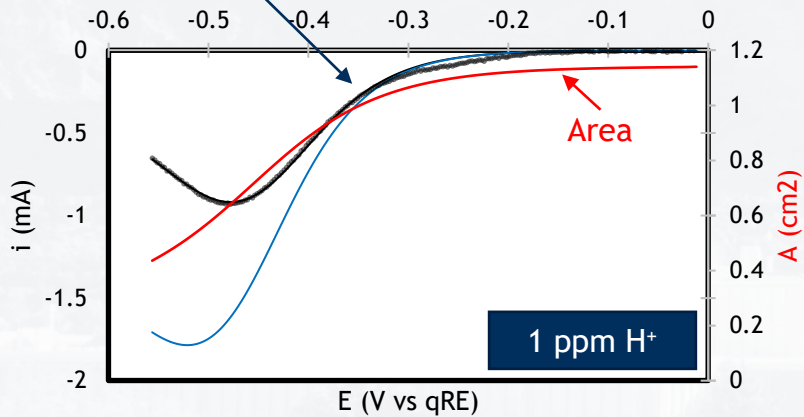


Safe handling of Cl₂ and HCl
gases

PyRO Lab

Develop sensors, models, and processes to support nuclear fuel processing, molten salt reactors, concentrated solar power, and other molten salt operations.

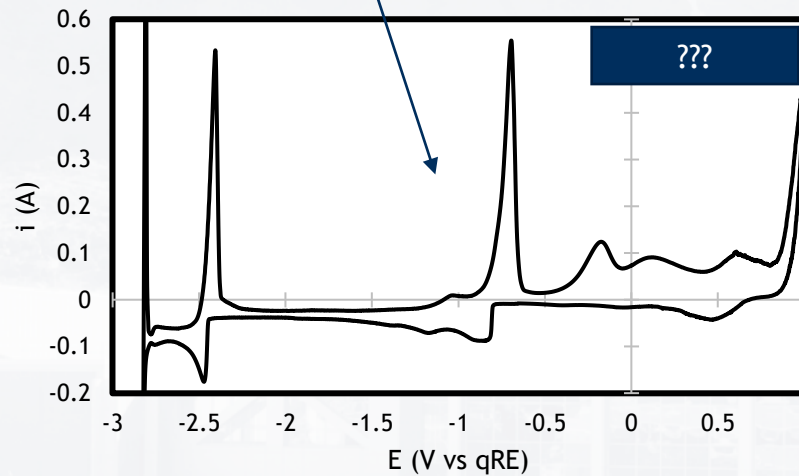
New Models:
Modified Randles-Ševčík
for insulating, gaseous
products



• Data — Model — RS — Area

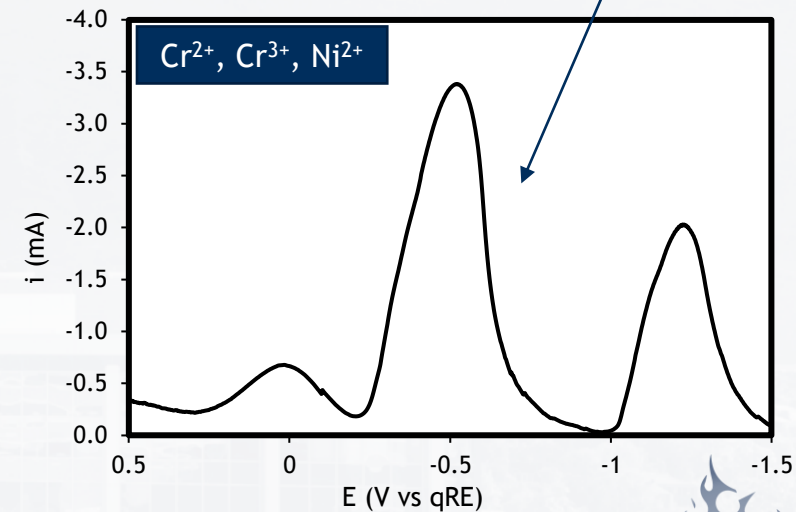
Low
Concentrations

Semi-Differentiation:
Splitting overlapping
peaks



Multiple Species

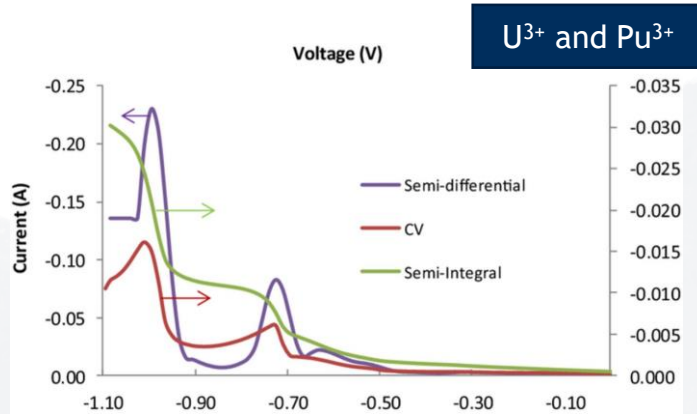
Thin-layer Electrodes:
Bulk electrolysis



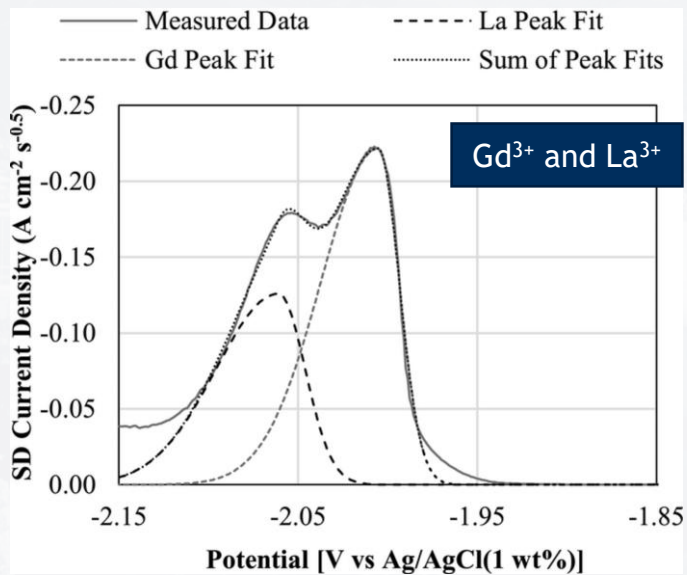
High
Concentrations



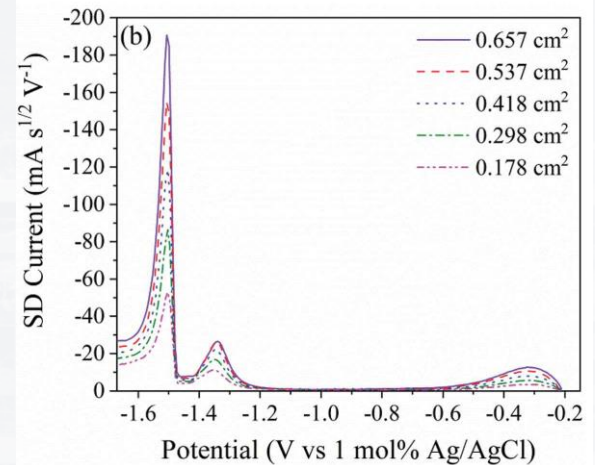
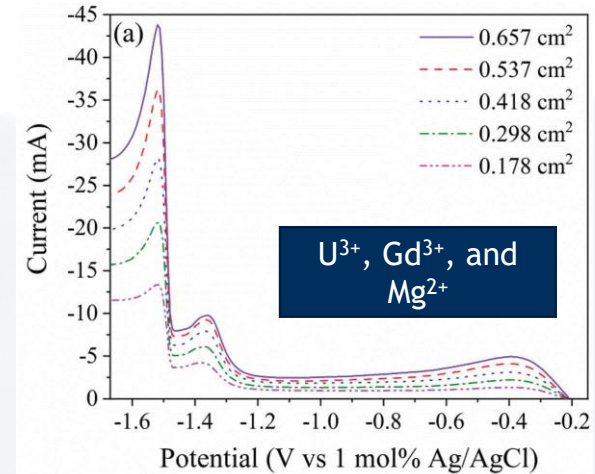
Motivation – Semi-Differentiation can Clarify



M. Tylka, J. Willit, J. Prakash, M. Williamson, *J. Electrochem. Soc.* **162**, H852 (2015)



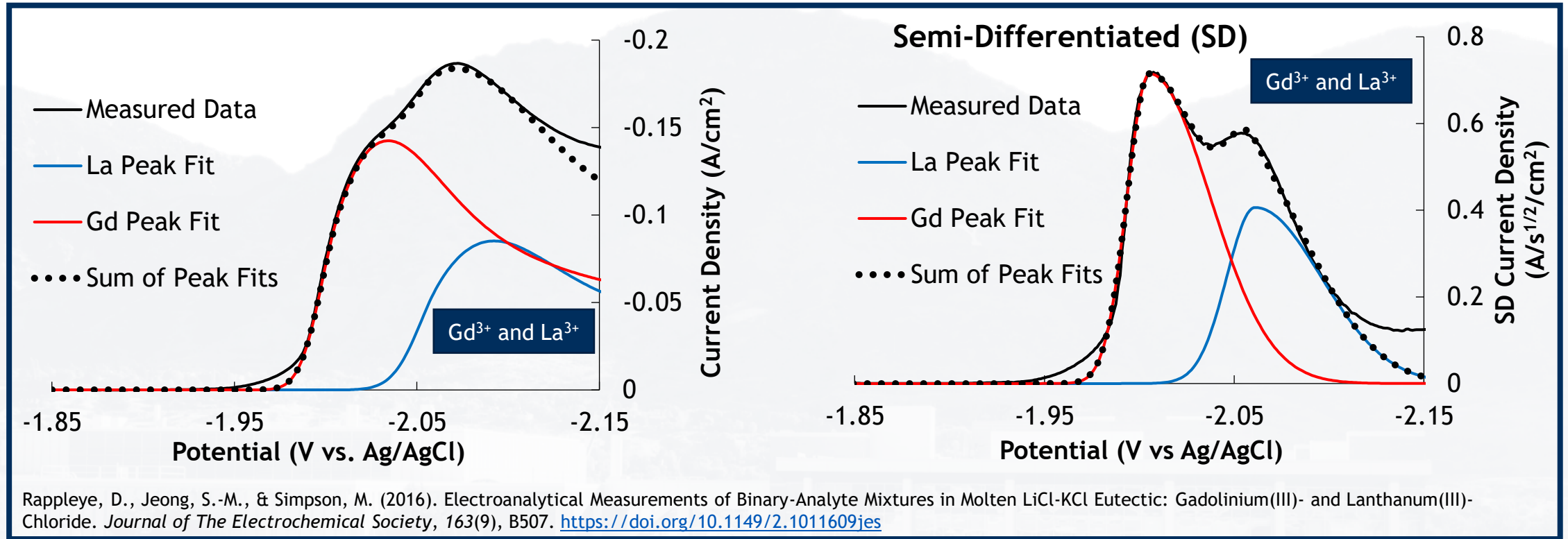
D. Rappleye, S.-M. Jeong, M. Simpson, *J. Electrochem. Soc.* **163**, B507 (2016)



H. Andrews & S. Phongikaroon, *Nucl. Tech.* **207**, 617-626 (2021)



Motivation – Peaks or Exponential Curves?



$$D^{1/2}i(t) = \frac{n^2 F^2 A \sqrt{D_o}}{2RT} v C_o e^{\frac{nF}{RT}(E_{eq} - vt - E_{1/2})}$$

Tylka, M. M., Willit, J. L., Prakash, J., & Williamson, M. A. (2015). Application of Voltammetry for Quantitative Analysis of Actinides in Molten Salts. *Journal of The Electrochemical Society*, 162(12), H852-H859. <https://doi.org/10.1149/2.0281512jes>

An exponential function???

Theory - What is a Semi-Derivative?

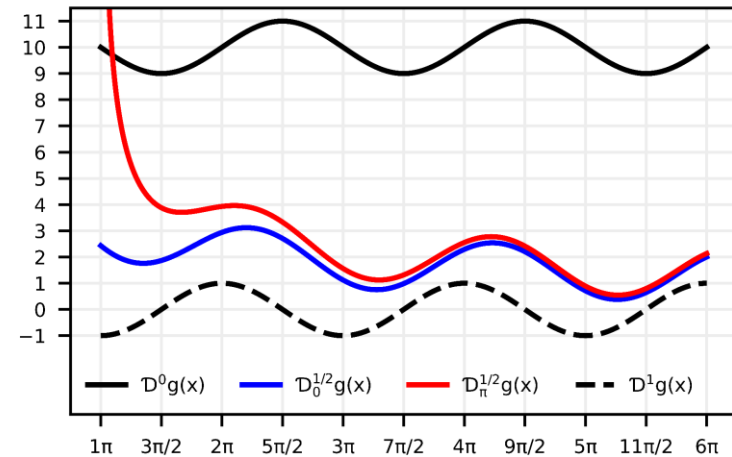
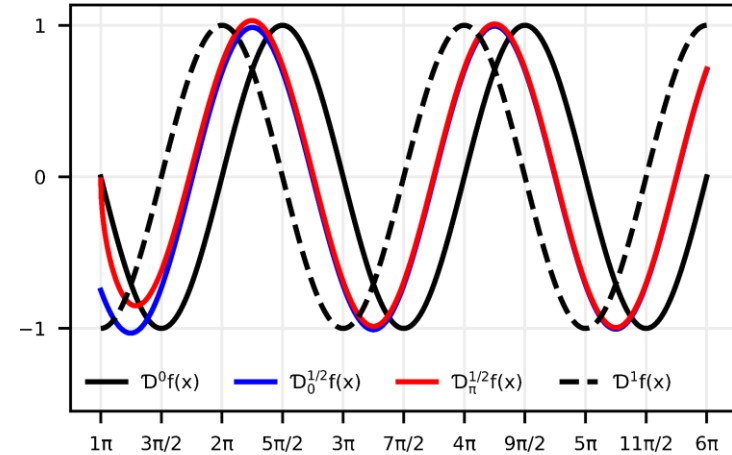
$$\frac{\partial^2 f(x)}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial f(x)}{\partial x} \right)$$

$$\frac{\partial f(x)}{\partial x}$$

$$f(x)$$

$$\frac{\partial^{-1} f(x)}{\partial x^{-1}} = \int f(x) dx$$

$$\frac{\partial^{-2} f(x)}{\partial x^{-2}} = \frac{\partial^{-1}}{\partial x^{-1}} \left(\frac{\partial^{-1} f(x)}{\partial x^{-1}} \right) = \iint f(x) dx dx$$



T. Williams, C. Vann, R. Fuller, D. Rappleye, *J. Electrochem. Soc.* 170, 042502 (2023)



Theory - What is a Semi-Derivative?

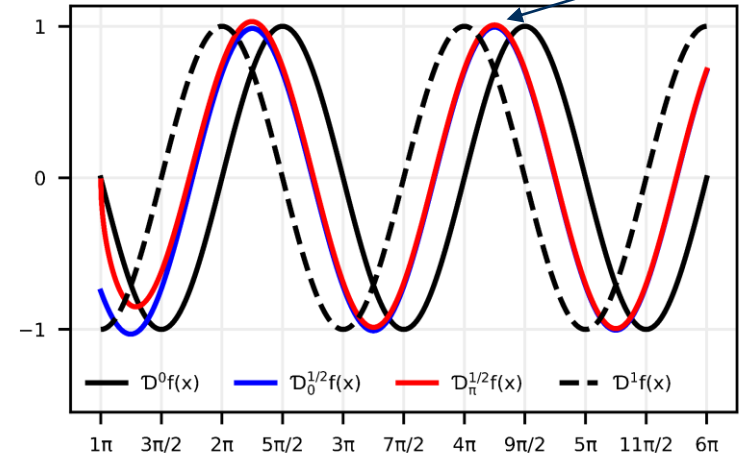
Half of the phase shift

$$\frac{\partial^2 f(x)}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial f(x)}{\partial x} \right)$$

$$\frac{\partial f(x)}{\partial x} \leftarrow \frac{\partial^{1/2} f(x)}{\partial x^{1/2}} = \mathcal{D}^{1/2} f(x)$$

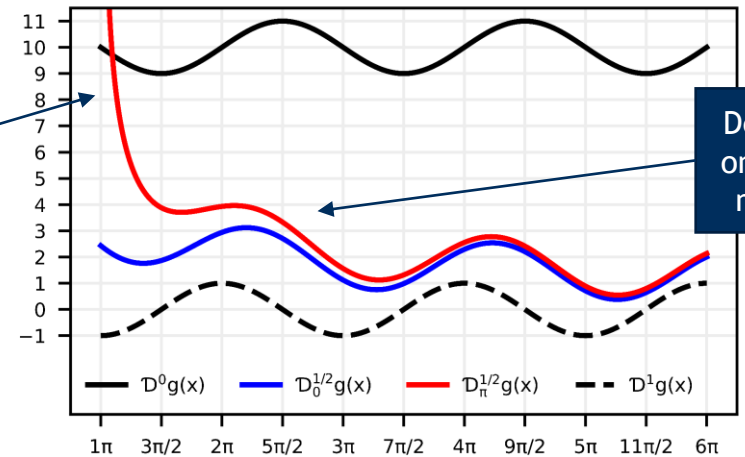
$$\frac{\partial^{-1} f(x)}{\partial x^{-1}} = \int f(x) dx$$

$$\frac{\partial^{-2} f(x)}{\partial x^{-2}} = \frac{\partial^{-1}}{\partial x^{-1}} \left(\frac{\partial^{-1} f(x)}{\partial x^{-1}} \right) = \iint f(x) dx dx$$



History dependence

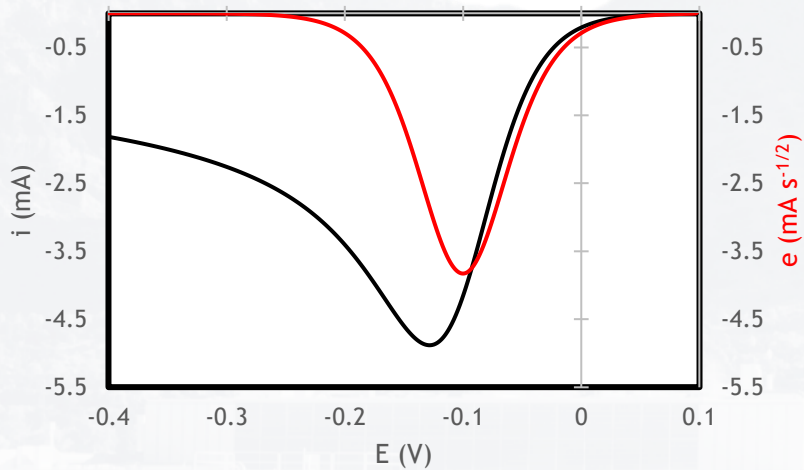
Dependence on slope and magnitude



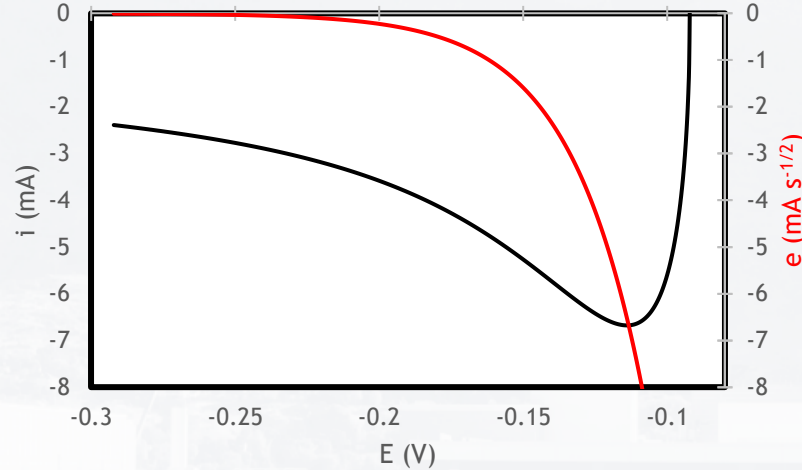
T. Williams, C. Vann, R. Fuller, D. Rappleye, *J. Electrochem. Soc.* 170, 042502 (2023)



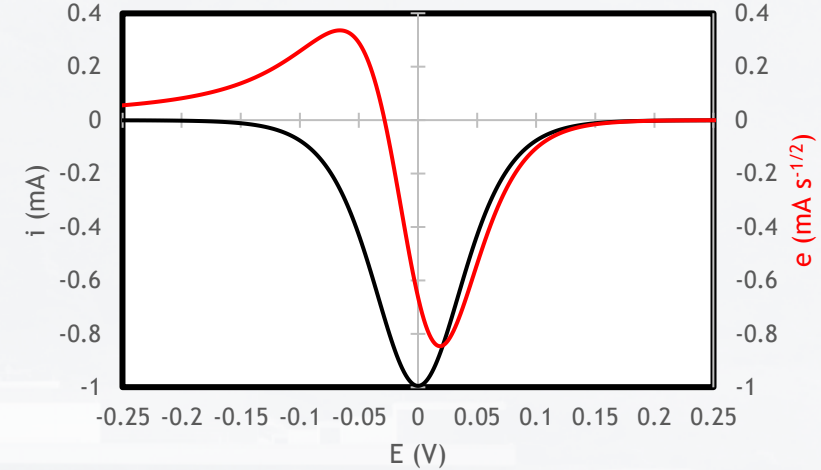
Theory – Semi-derivatives of Common Curves



Randles-Ševcík



Berzins-Delahay



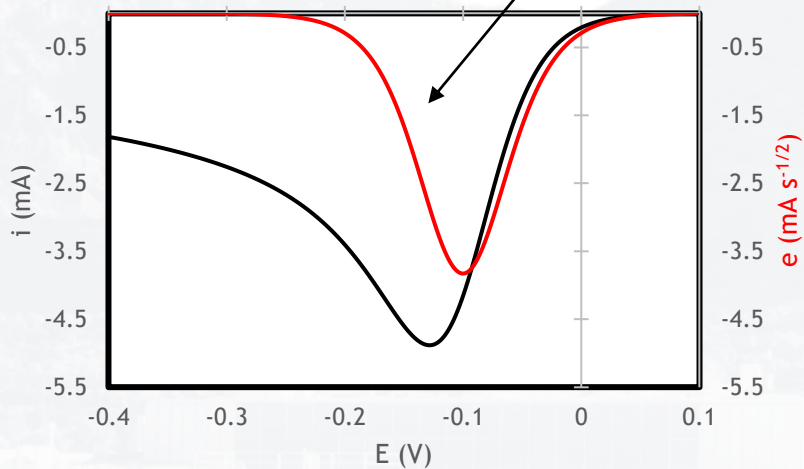
Thin-layer CV

Hubbard and Anson, *Electroanalytical Chemistry*, Vol. 4, pg 133

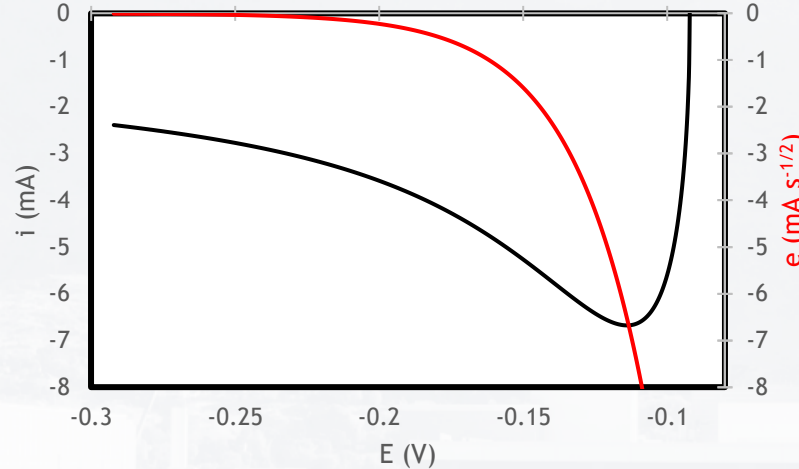


Theory – Semi-derivatives of Common Curves

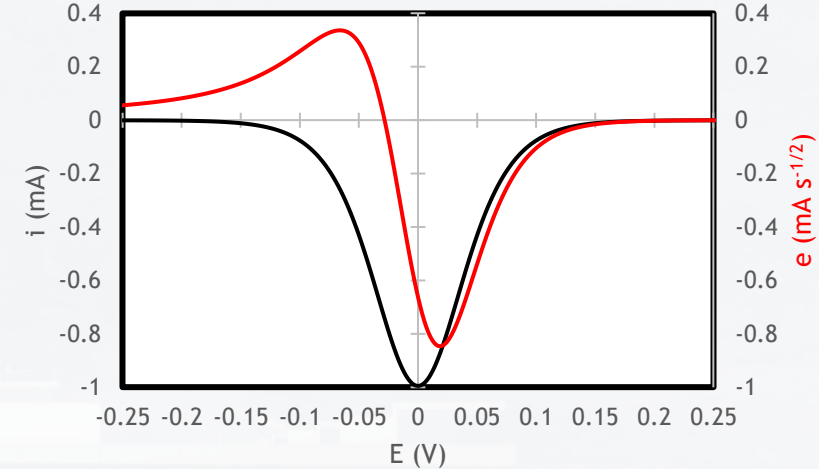
$$e(t) = -\frac{n^2 F^2 A C_0^* \nu}{4RT} D_0^{1/2} \operatorname{sech}^2 \left(\frac{nF}{2RT} (E(t) - E_{1/2}) \right)$$



Randles-Ševc'ik



Berzins-Delahay



Thin-layer CV

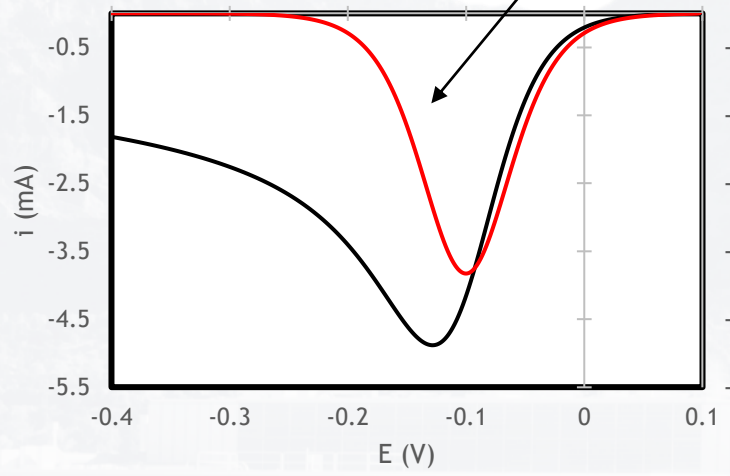
Hubbard and Anson, *Electroanalytical Chemistry*, Vol. 4, pg 133



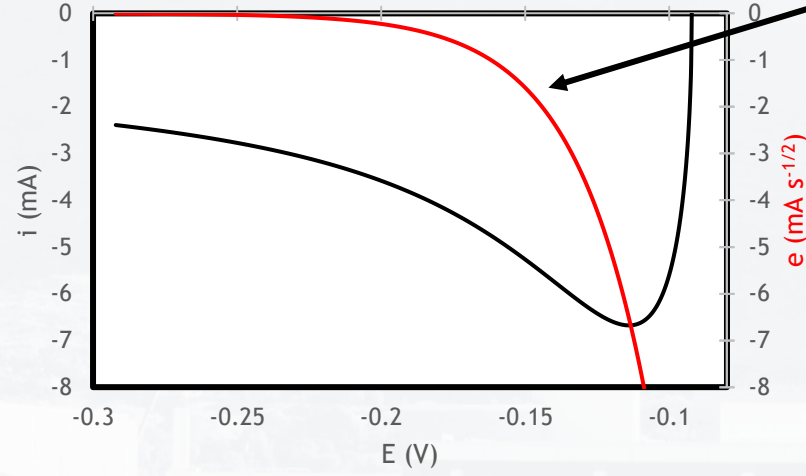
Theory – Semi-derivatives of Common Curves

$$e(t) = -\frac{n^2 F^2 A C_0^* \nu}{4RT} D_0^{1/2} \operatorname{sech}^2 \left(\frac{nF}{2RT} (E(t) - E_{1/2}) \right)$$

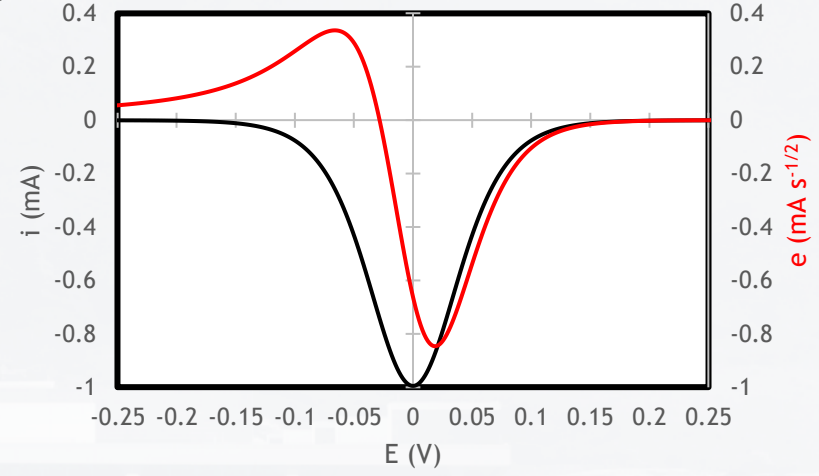
$$e(t) = -\frac{n^2 F^2 A C_0^* \nu}{RT} D_0^{1/2} \exp \left(\frac{nF}{RT} (-vt) \right)$$



Randles-Ševcík



Berzins-Delahay



Thin-layer CV

Hubbard and Anson, *Electroanalytical Chemistry*, Vol. 4, pg 133

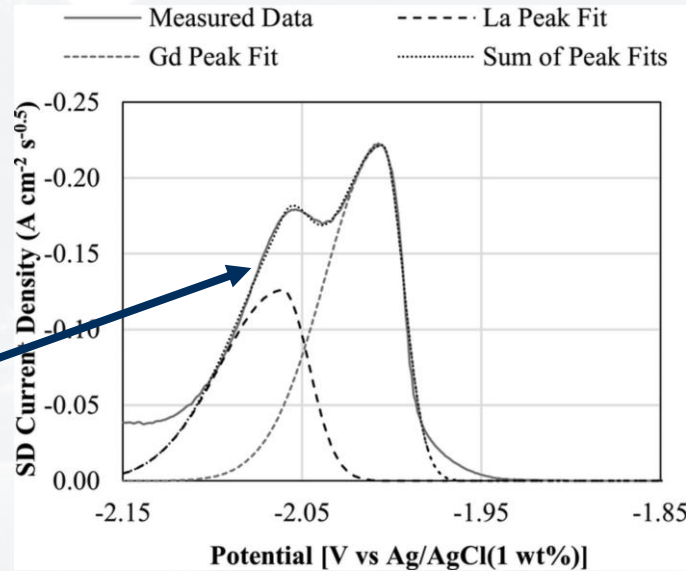


P. Dalrymple-Aford, M. Goto, K.B. Oldham, *J. Electroanal. Chem. Interfacial Electrochem.* **85**, 1 (1977) and T. Williams, R. Fuller, C. Vann, D. Rappleye, *J. Electrochem. Soc.*, **170**, 042502 (2023)

Results – Reconciling Peaks and Exponentials

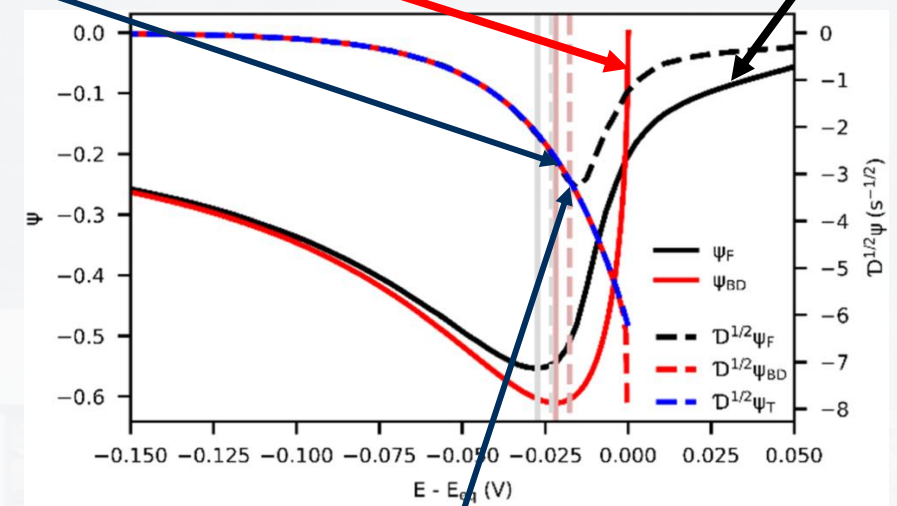
$$e(E_p) = -0.4257 \frac{n^2 F^2 A C_0^* \nu}{RT} D_0^{1/2}$$

Deposition Rxns



D. Rappleye, S.-M. Jeong, M. Simpson, *J. Electrochem. Soc.* 163, B507 (2016)

Theoretical ideal deposition model
 Theoretical non-ideal deposition model



T. Williams, R. Fuller, C. Vann, D. Rappleye, *J. Electrochem. Soc.*, 170, 042502 (2023)

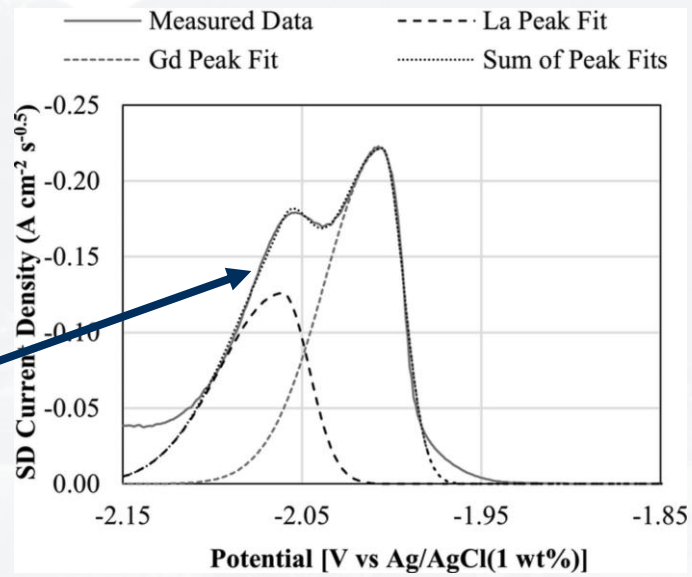
$$e(E_{1/2}) = - \frac{n^2 F^2 A C_0^* \nu}{2RT} D_0^{1/2}$$



Results – Reconciling Peaks and Exponentials

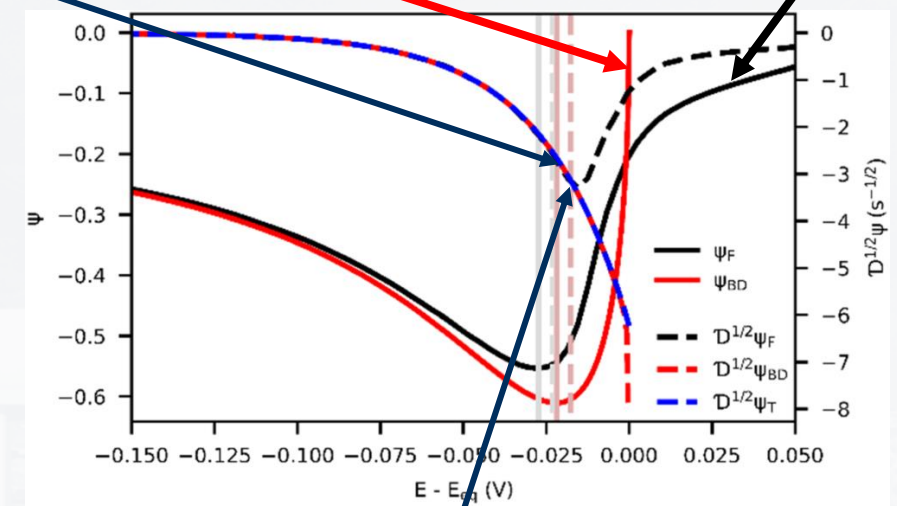
$$e(E_p) = -0.4257 \frac{n^2 F^2 A C_0^* \nu}{RT} D_0^{1/2}$$

Deposition Rxns



D. Rappleye, S.-M. Jeong, M. Simpson, *J. Electrochem. Soc.* 163, B507 (2016)

Theoretical ideal deposition model
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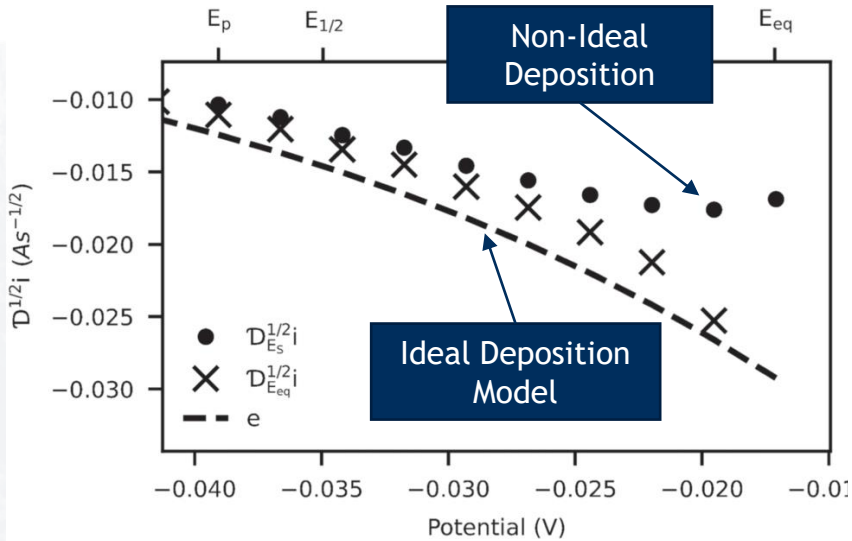
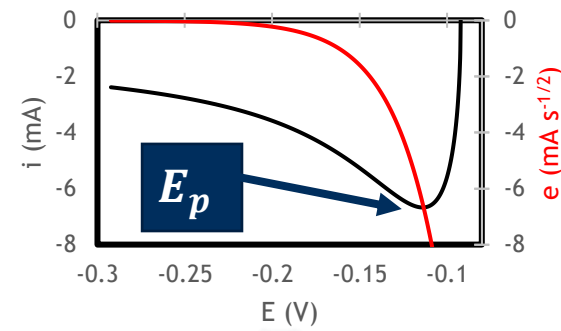
T. Williams, R. Fuller, C. Vann, D. Rappleye, *J. Electrochem. Soc.*, 170, 042502 (2023)

Non-ideal Deposition Model Needed

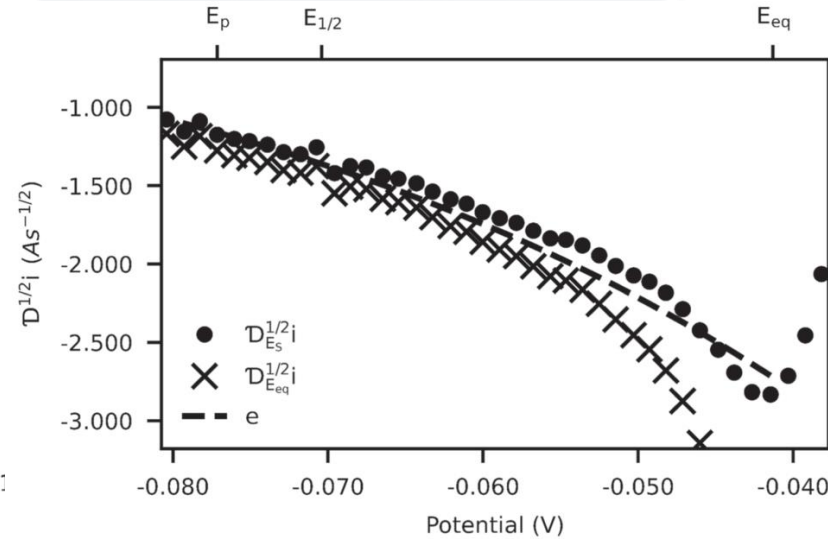
$$e(E_{1/2}) = - \frac{n^2 F^2 A C_0^* \nu}{2RT} D_0^{1/2}$$



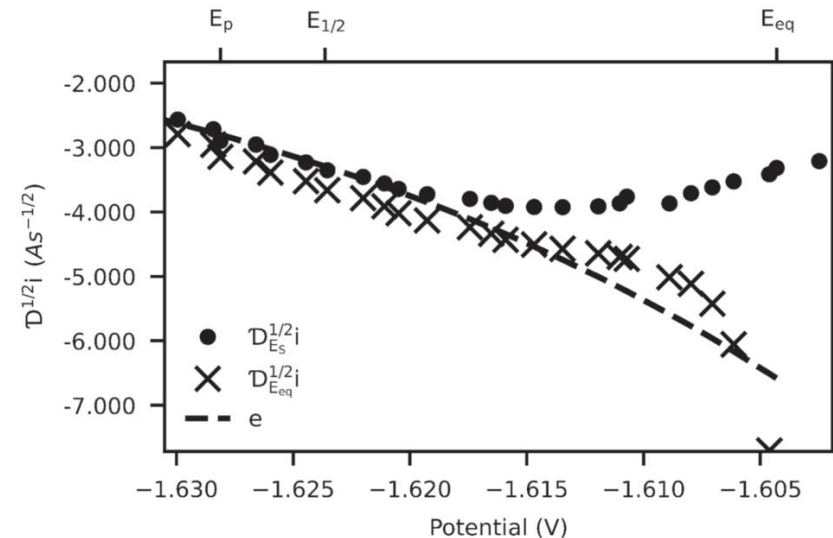
Results – Model vs Data (Curves)



0.027 M AgNO_3 in 1 M HNO_3
298 K
300 mV/s



0.42 wt% NiCl_2 in LiCl
974 K
1000 mV/s



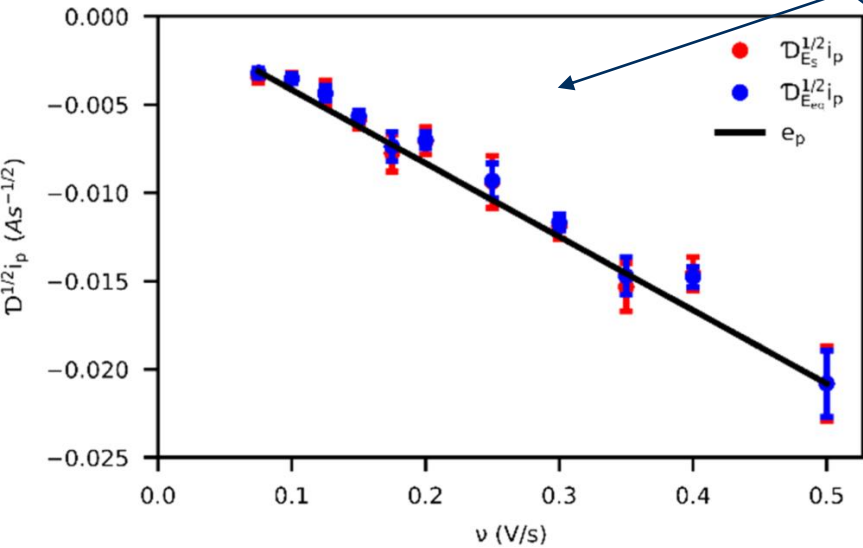
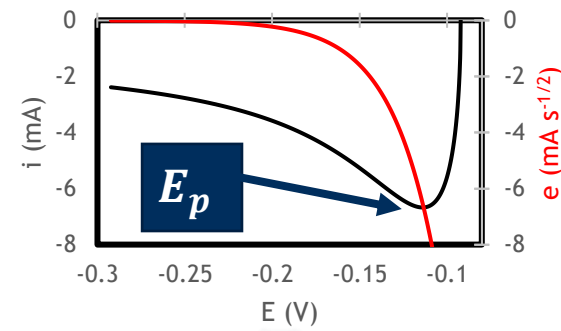
0.43 wt% LaCl_3 in LiCl
971 K
2000 mV/s

Theory and data converge at $E_{1/2}$,
best at E_p

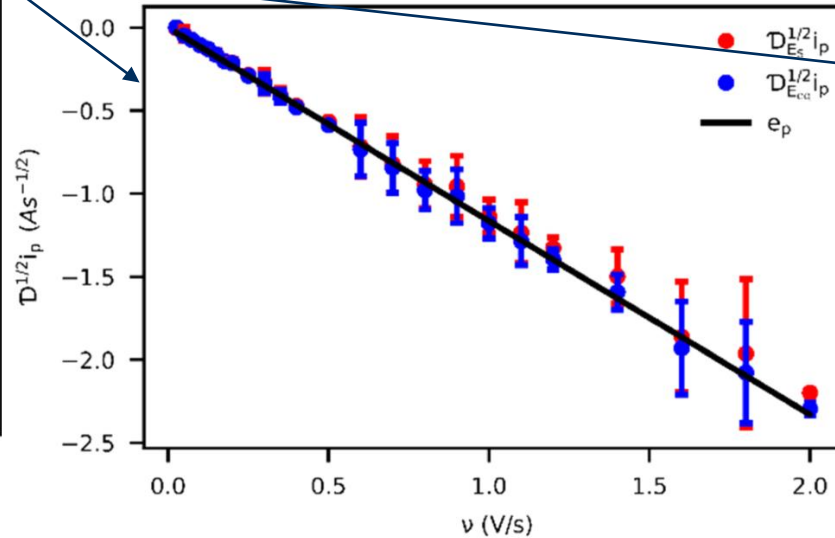


Results – Model vs Data (E_p)

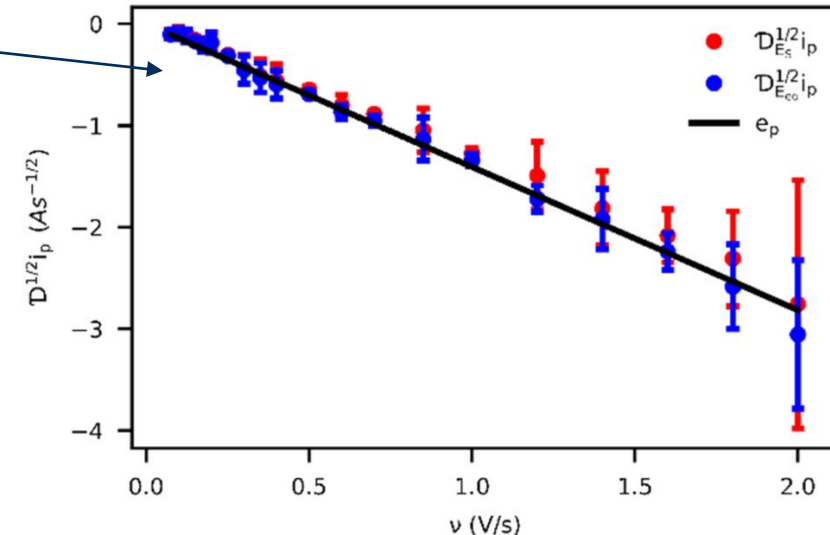
Linear i_p vs $v^{1/2}$



0.027 M AgNO_3 in 1 M HNO_3
298 K
300 mV/s



0.42 wt% NiCl_2 in LiCl
974 K
1000 mV/s



0.43 wt% LaCl_3 in LiCl
971 K
2000 mV/s

$$e(E_p) = -0.4257 \frac{n^2 F^2 A C_o^* \nu}{RT} D_o^{1/2}$$

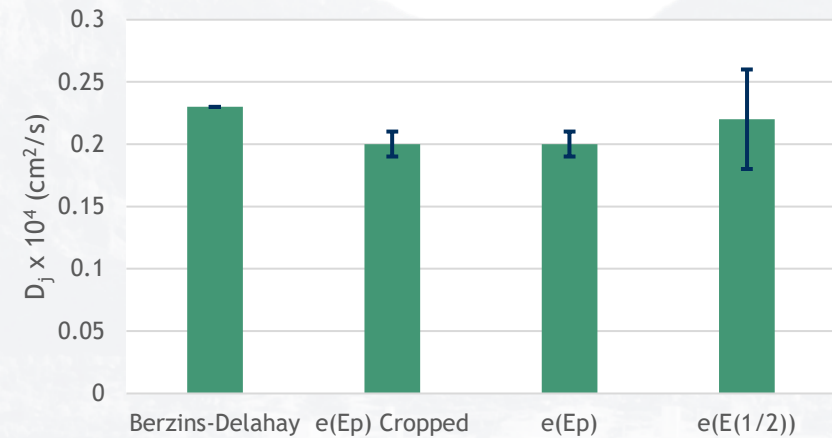


Results – Diffusion Coefficient Calculations

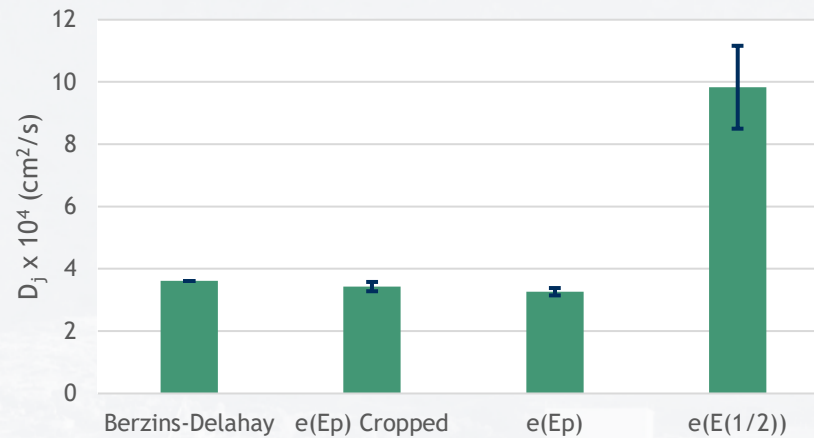
$$i(E_p) = -0.61AC_0^* \left(\frac{(nF)^3 v}{RT} \right)^{1/2} D_0^{1/2}$$

$$e(E_p) = -0.4257 \frac{n^2 F^2 AC_0^* v}{RT} D_0^{1/2}$$

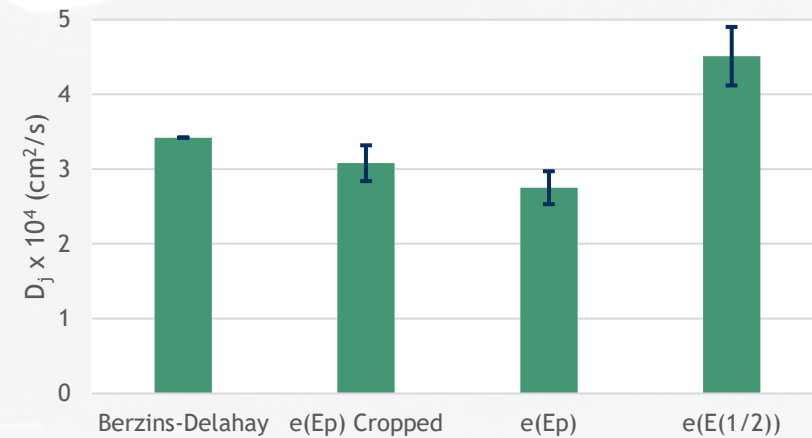
$$e(E_{1/2}) = -\frac{n^2 F^2 AC_0^* v}{2RT} D_0^{1/2}$$



0.027 M AgNO₃ in 1 M HNO₃
298 K
300 mV/s



0.42 wt% NiCl₂ in LiCl
974 K
1000 mV/s



0.43 wt% LaCl₃ in LiCl
971 K
2000 mV/s

Conclusions & Next Steps

Conclusions:

- Semi-derivatives (SD) can help separate data.
- SD peaks are attributed to nucleation processes.
- The derived relations are as analytically useful as the Berzins-Delahay relations.

Next Steps:

- Develop non-ideal deposition models for cyclic voltammetry.
- Investigate the limits of how successful overlapping peaks can be separated.
- Optimize the fractional differentiation order. No reason why a semi-derivative would be necessarily best.



